Quarta Lista de Exercícios

Questão 1:
Consider a queueing system with one server representing, for instance, an automatic teller machine (ATM) (caixa eletrônico). Customers arrive at the ATM according to a Poisson process with rate $\lambda$. Let $s$ be the random variable equal to the amount of time a customer spends using the ATM and $E[s]$ the expected value.

For this problem assume $\lambda = 0.3$ customers per minute and $E[s] = 3$ minutes.

1. For this system indicate the state variable you will use and draw the state transition diagram.

2. Show $Q$, the state transition rate matrix (also called the generator matrix) for this system.

Let $\pi_i$ be the fraction of time the system spends in a state with $i$ customers in system.

1. Now assume that a friend told you that $\pi_i = (1 - \rho)^i$ where $\rho = \lambda E[s]$, so you would not need to lose time taking measurements. Calculate $N$, the expected number of customers in system from the formula above. (EXPLAIN your answer.)

2. Calculate the $W$, the expected waiting time in system.

3. Calculate the system utilization.

Questão 2:
Consider the same automatic teller machine (ATM) (caixa eletrônico) above, with a single ATM. However, in this system customers are impatient, so if they arrive and find more than 3 customers in line (including the one using the machine) they leave immediately.

For this problem assume $\lambda = 0.3$ customers per minute and $E[s] = 3$ minutes, as in previous question.

- Obtain the expected number of customers in queue and compare with the expected number calculated when there are no impatient customers (question above).
- Obtain the average waiting time.
• Calculate the rate at which customers depart when they find a big line.

• Calculate the rate at which customers that use the teller machine depart from the system.

• Calculate the system utilization.

**Questão 3:**

Now the bank will give preference to the elderly customers. That is, the elderly customers (those that are at least 60 years old) wait in a separate line and, as soon as someone leaves the single ATM, if there is an old person waiting in line, he/she is the next to be served. Customers are impatient, as in the previous problem, and they leave the system if there are more than 3 customers in the system (considering both lines and the person in service).

Assume that 10% of the arriving customers are over 60 years of age. Furthermore, the expected service time of an older person is 50% more than for the remaining customers.

• For this system indicate the state variables you will use and draw the state transition diagram. THIS IS VERY IMPORTANT. PAY ATTENTION.

• Show Q, the state transition rate matrix (also called the generator matrix) for this problem.

• Obtain the expected number of customers in the normal and elderly lines.

• Obtain the average waiting time for each customer class, and the overall expected waiting time.

• Calculate the system utilization.

**Questão 4:**

Bank B manages their customer lines as follows: When the first customer arrives, only one teller is available. However, as soon as the line is greater than 4 customers, another teller is called for servicing customers. When more than 6 customers are in line, a third teller starts helping the other two. The third teller stops helping when the line decreases to 5 and the second stops when there are only 2 customers waiting. (In this problem assume that “customers in line” includes only those waiting for service, and not customers being served.)

Assume at most 10 customers can be in line, that is, if a new customer arrives and find 10 customers waiting for service he/she gives up and departs. Also assume the arrival rate \( \lambda = 0.33 \) customers per minute and the expected service time \( E[s] = 3 \) minutes.

• Choose the state variable(s), indicating the meaning of each and draw the Markov chain for this system.

• Calculate the loss rate as a function of \( \pi_i \).

• Calculate the expected time customers wait until they start being served.
**Questão 5:**

An amateur pilot visits several cities in Brazil. She spends 5 days in each city visited, and randomly chooses (with equal probability) the next city to be visited, for those that are possible to reach from the city the pilot is visiting. (Due to flight restrictions, the pilot cannot fly to any city from the one she is currently visiting.)

Assume that the pilot visits Rio, São Paulo (SP), Belo Horizonte (BH), Vitória and Brasília only. Further, assume that it is possible to fly between:

- Rio ↔ SP, Rio ↔ BH, and Rio ↔ Vitória;
- SP ↔ BH, SP ↔ Brasília;
- BH ↔ Vitória, BH ↔ Brasília.

In addition, assume that the flying time is irrelevant with respect to the time spent in a city.

1. Show how to calculate the conditional rate from Rio to São Paulo. First define this quantity (consult the class notes).
2. Do every step necessary to calculate the fraction of time the pilot spends in each city.