10.3 HYPOTHESIS TESTING

Suppose now that we wish to reduce the value of \( \beta \) to 0.1 for \( \lambda = 0.0002 \). This can be done either by increasing the sample size (in this case, the number of observed failures) or by allowing a larger value of \( \alpha \):

\[
C = \frac{\chi^2_{2\alpha, \lambda_0}}{2\lambda_0} = \frac{\chi^2_{2\alpha, \lambda}}{2\lambda}.
\]

So

\[
\chi^2_{2\alpha, \lambda_0} = \frac{0.00025}{0.00002} \chi^2_{2, 0.9}.
\]

We can now specify either \( \alpha \) or the sample size \( r \); should we choose \( r = 40 \), we get

\[
\chi^2_{2\alpha, \lambda_0} = 1.25 \cdot 64.25 = 80.35.
\]

Since \( \chi^2_{0.5} = 79.33 \), the value of \( \alpha \) is approximately 0.5. Of course, we could choose smaller \( \alpha \) at the expense of further increase in the sample size.

Problems

1. To test whether a given circuit is fault-free, we drive it for a sequence of 100 inputs and observe 37 ones at the output (63 zeros). If the circuit is fault-free, 50 ones are expected. At a significance level of 0.05 (probability of a false alarm), can we reject the hypothesis that the circuit is fault-free? Compute the descriptive level of the test.

2. In statistical pattern recognition, one method used to distinguish the letter B from the numeral 8 is to compute the straightness ratio, defined as a value of the random variable \( X \), which is the ratio of the symbol's height to its arc length (on the left-hand side), and perform a hypothesis test on it. Suppose that the conditional distribution of \( X \) given that the symbol is 8 is normal with mean 0.8 and variance 0.01, while the conditional distribution of \( X \) given B is normal with mean 0.96 and variance 0.01. The pattern recognition problem is now cast as a hypothesis testing problem:

\[
H_0 : E[X] = 0.8 \quad \text{versus} \quad H_1 : E[X] = 0.96.
\]

Suppose after measurement of the given symbol we reject \( H_0 \) if \( \bar{x} > 0.90 \). Compute the error probabilities \( \alpha \) and \( \beta \).

3. Consider the combinational circuit in problem 1 of the review problems for Chapter 1. First compute the probability of a 1 at the output, assuming that at each of the inputs the probability of a 1 is \( \frac{1}{2} \). Then test the hypothesis that the circuit is fault-free versus the hypothesis that there is a stuck-at-0 type fault at input \( x_1 \). For this case compute the probability of false alarm (\( \alpha \)) and the probability of escape (\( \beta \)), assuming the length of test \( n = 400 \) and test stringency \( \epsilon = 0.005 \). Repeat the calculation of \( \beta \) for each of the remaining 13 fault types.

4. In selecting a computer server we are considering three alternative systems. The first criterion to be met is that the response time to a simple editing command should be less than 3 s at least 70% of the time. We would like the type I error probability to be less than 0.05. On \( n = 64 \) randomly chosen requests the number \( m \) of requests that met the criterion of \( < 3 \) s response were found to be as shown in the following table: