

Estatística e Modelos Probabilísticos - COE241

Aula de hoje

Propriedades da média e da variância

Covariância, coeficiente de correlação

Esperança da soma de variáveis aleatórias

- If $Z=X+Y$, then
 - $E[X+Y] = E[X]+E[Y]$
 - For the above to hold, X, Y need not be independent
- More generally, Linearity property of expectation (no need to assume independence)

$$E\left[a_0 + \sum_{i=1}^n a_i X_i\right] = a_0 + \sum_{i=1}^n a_i E[X_i]$$

Esperança do produto de variáveis aleatórias

- If $Z=XY$, then

- $E[XY] = E[X]E[Y]$ (if X, Y are mutually independent)

- More generally

$$E \left[\prod_{i=1}^n X_i \right] = \prod_{i=1}^n E[X_i] \quad (X_i \text{ mutually independent})$$

$$E \left[\prod_{i=1}^n \phi_i(X_i) \right] = \prod_{i=1}^n E[\phi_i(X_i)] \quad (X_i \text{ mutually independent})$$

Variância: outra expressão

$$\begin{aligned}\sigma^2 &= E[(X - E[X])^2] \\ &= E[X^2 - 2XE[X] + (E[X])^2] \\ &= E[X^2] - E[2XE[X]] + (E[X])^2 \\ &= E[X^2] - 2E[X]E[X] + (E[X])^2\end{aligned}$$

(noting that $E[X]$ is a constant). Thus

$$\sigma^2 = E[X^2] - (E[X])^2$$

Variância: outra expressão

$$\sigma^2 = E[X^2] - (E[X])^2$$

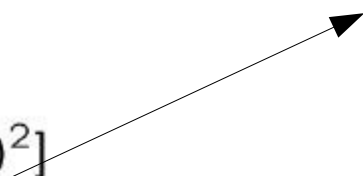
This formula for $Var[X]$ is usually preferred over the original definition:

$$Var[X] = \mu_2 = \sigma_X^2 = \begin{cases} \sum_i (x_i - E[X])^2 p(x_i) & \text{if } X \text{ is discrete} \\ \int_{-\infty}^{\infty} (x - E[X])^2 f(x) dx & \text{if } X \text{ is continuous} \end{cases}$$

Variância da soma de variáveis aleatórias

- - $Var[X+Y] = Var[X] + Var[Y]$
(If X, Y independent rv's)

Variância da soma de variáveis aleatórias

$$E[((X - E[X]) + (Y - E[Y]))^2]$$


$$\begin{aligned} \text{Var}[X + Y] &= E[((X + Y) - E[X + Y])^2] \\ &= E[((X + Y) - E[X] - E[Y])^2] \\ &= E[(X - E[X])^2 + (Y - E[Y])^2 + 2(X - E[X])(Y - E[Y])] \\ &= E[(X - E[X])^2] + E[(Y - E[Y])^2] + 2E[(X - E[X])(Y - E[Y])] \\ &= \text{Var}[X] + \text{Var}[Y] + 2\text{Cov}(X, Y) \end{aligned}$$

where

$$\begin{aligned} \text{Cov}(X, Y) &= E[(X - E[X])(Y - E[Y])] \\ &= E[XY - YE[X] - XE[Y] + E[X]E[Y]] \\ &= E[XY] - E[X]E[Y] = 0 \text{ (if } X \text{ and } Y \text{ are independent)} \end{aligned}$$

Variância da soma de variáveis aleatórias

$$\text{Var} \left[\sum_{i=1}^n a_i X_i \right] = \sum_{i=1}^n a_i^2 \text{Var}[X_i] \text{ (if } X_i\text{'s independent)}$$

Dependência entre variáveis aleatórias

- $\text{Cov}(X, Y) \stackrel{\text{def}}{=} E[(X - E[X])(Y - E[Y])]$
- $\text{Cov}(X, Y) = 0$ then X and Y are uncorrelated
- If X, Y independent then $\text{Cov}(X, Y) = 0$
- Se $\text{Cov}(X, Y) = 0$, não significa que X e Y são independentes
- Covariância expressa a dependência linear

Dependência entre variáveis aleatórias: exemplo

- Seja X v.a. Uniforme $(-1,1)$ e $Y=X^2$, logo Y é dependente de X . Qual o valor de $\text{Cov}(X,Y)$?

$$\text{Cov}(X, Y) = E[XY] - E[X]E[Y] = 0$$

- Y é dependente de X , mas X e Y são não correlacionadas.

Coeficiente de Correlação

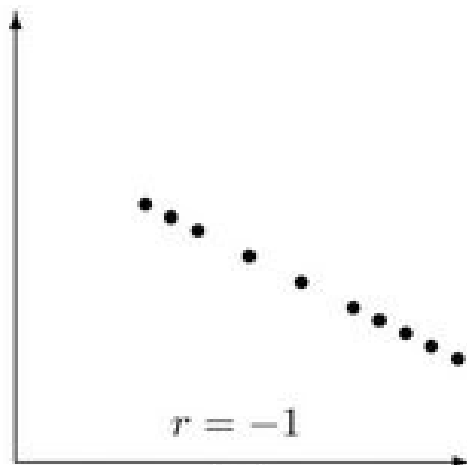
- (Cross) Correlation Co-efficient:

$$\begin{aligned}\rho(X, Y) &= \frac{Cov(X, Y)}{\sqrt{Var[X]Var[Y]}}, \quad -1 \leq \rho(X, Y) \leq 1 \\ &= \frac{Cov(X, Y)}{\sigma_X \sigma_Y}\end{aligned}$$

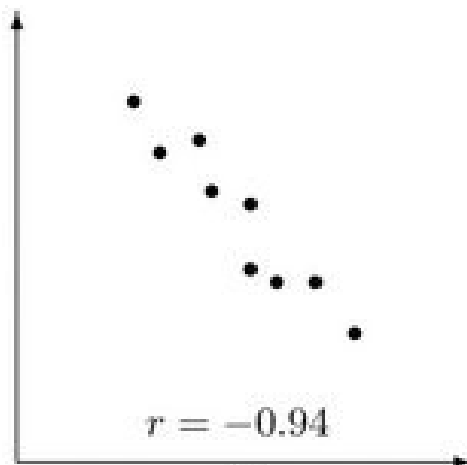
- The closer $\rho(X, Y)$ is to -1 or 1 , the stronger the linear relationship is between X and Y .

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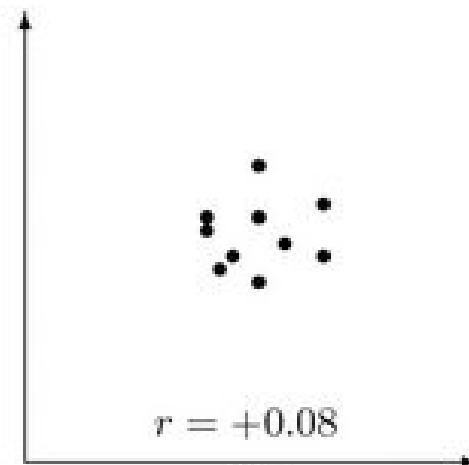
Coeficiente de Correlação



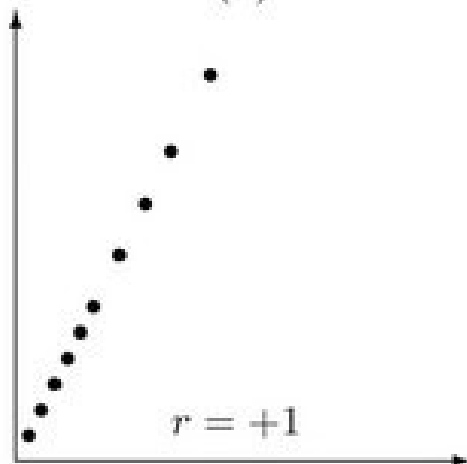
(a)



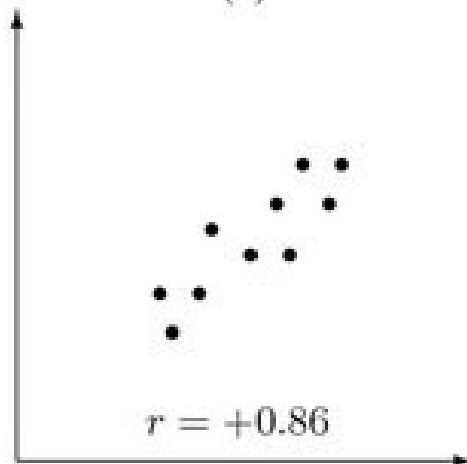
(b)



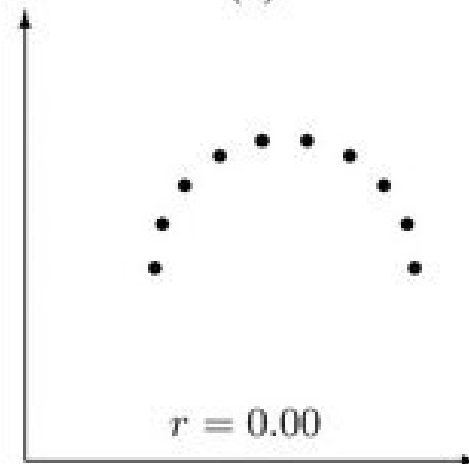
(c)



(d)



(e)



(f)